

Autostochasticity and conversion of lasing fluctuations in a ring cavity with a nonlinear element

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Abstract

A statistical theory for the evolution of radiation in a cavity excited by a partially coherent signal is constructed based on a simple theoretical model for a ring cavity with a nonlinear element, leading to a difference equation with a rapidly oscillating exponential. It is shown that mixing, being the cause of the onset of deterministic autostochastic movement (chaos) for strictly coherent excitation, leads to conversion of the statistical characteristics of the signal in the presence of fluctuations: in particular, noise intensity rises due to lowering of the intensity of the coherent component (the system behaves a noise generator). An approximate method for calculation of statistical parameters of the generated noise signal is proposed .

The evolution of optical field in a ring cavity (RC) with a nonlinear medium can be described (in the plane-wave approximation) by a differential equation with retarded argument; it can be reduced to a difference equation (the Ikeda model) [1]-[6], if certain additional conditions are satisfied. It is now well known that this type of equations can have solutions, corresponding to complex motion with unstable trajectories, including periodic self-oscillations, chaotic motion with trajectory, tending asymptotically to a singular attractor and others. Certain types of regular and chaotic self-oscillations were observed in similar systems in numerical simulations [1]-[8] and experimentally [9]-[11].

We must note that the problem of the fluctuations influence is essential for analysis of the nonlinear systems dynamics (in particular, in chaotic and near-chaotic regimes); the very possibility of observing one or the other type of dynamics is determined by its stability with respect to small random perturbations. In particular, fluctuations can rough a fine structure of a chaotic attractor and limit the number of observable period-doubling bifurcations [12]. It is also important that such systems may amplify noisy signal and transform its statistical characteristics in a nontrivial manner; the latter allows us to build a noise generator, synthesizing noise signal with nonstationary properties.

This paper is devoted to the study of transformation of fluctuations in a RC with a nonlinear element. We consider a simple RC model, described by a difference evolution equation:

$$E(t) = 1 + \xi(t) + \kappa E(t - T_r) \exp(i\lambda |E(t - T_r)|^{2k} + i\theta_0), \quad (1)$$

(we omit the procedure of deriving Eq. (1) from physical equations, since it is presented rather completely in [1]-[6] in different versions, and has also been reproduced in [7] and [8]). The variables in Eq. (1) have the meaning of slowly varying electric field amplitudes of the light wave; $E(t)$ is the amplitude at the entrance to the nonlinear medium, $\xi(t)$ is the noisy component of the exciting signal (NCE) (the amplitude of the coherent component of this signal is constant

and normalized to unity). T_r is round-trip time of the RC, κ is the intensity loss factor per pass in the RC ($\kappa < 1$). If the RC contains a two-level medium, $k = 1$ for one- and two-photon transitions; $k = s - 1$ for s -photon transitions [6]. We limit our treatment of Eq. (1) for $\lambda \gg 1$, as was shown in [6], dynamics with strong mixing sensitive to noise arises under this condition. The case $\lambda \gg 1$ is interesting first of all for its relative simplicity from the computation point of view; we can find the statistical characteristics of the noisy radiation component (with certain additional assumptions about the NCE statistics) analytically. It is convenient instead of Eq. (1) it is convenient to consider evolution mapping

$$Z_{N+1} = \xi(t) + F(Z_N) \equiv 1 + \xi(t) + \kappa Z_N \exp(i\lambda |Z_N|^{2k} + i\theta_0), \quad (2)$$

which is derived from (1) by substituting $Z_N = E(t_0 + NT_r)$, $\xi_N = \xi[t_0 + (N+1)T_r]$; it describes evolution in a discrete time scale. Let us first consider the case $T_r = \tau_c \gg \tau^*$, where τ_c is the correlation time of NCE (being a stationary random process) and τ^* is the characteristic time for transitional processes in the nonlinear element ($\tau^* \sim T_1, T_2$ for the RC with a two-level medium). In view of the assumption made, the exciting wave and the wave travelling around the RC are statistically independent; evolution of the random amplitude can be described using the Kolmogorov-Chapman equation

$$P_{N+1}(X) = \int \int dY dZ P_n(X - Y) W(Y, Z) P_N(Z), \quad (3)$$

where $P_N(X)$ is the amplitude distribution density at the time $t_0 + NT_r$; $P_n(X)$ is the distribution density of each of the independent random quantities $\xi_r, r = 1, 2, 3, \dots$; $W(Y, Z) = \delta^{(2)}(Y - F(Z))$; and $\delta^{(2)}(X) = \delta(\text{Re}X)\delta(\text{Im}X)$ is a complex delta-function. Assuming $\lambda \gg 1$, we use the random phases approximation

$$W(Y, Z) \rightarrow \tilde{W}(Y, Z) = \langle \delta^{(2)}(Y - 1 - \kappa Z e^{i\theta}) \rangle_{\text{phase}} = (2\pi |Y - 1|)^{-1} \delta(|Y - 1| - \kappa |Z|) \quad (4)$$

Henceforth $\langle \dots \rangle_{\text{phase}}$ denotes averaging over the random phase angle (over several angles) with uniform distribution density in $[0, 2\pi]$ [over angle θ in Eq. (4)].

The possibility of using the approximation (4) was discovered in [6]; omitting a detailed account, we will repeat here some qualitative reasons. Considering the Fourier transform of Eq. (3), we can show that application of approximation (4) leads to negligence by terms responsible for forming the small-scale structure of the kernel W and distributions P_N . This procedure can be justified in the presence of a noisy component in the exciting radiation; in the large λ case, even low intensity noise can lead to roughening of the distribution and loss of information about the small-scale structure. As it was noted in [6], we should distinguish the cases: (a) NCE intensity is low; it is sufficient for roughening of the distribution and generated noisy radiation in the RC, but the statistics of the latter is independent of the statistical properties of the NCE; (b) NCE intensity is comparatively high, so that the statistics of the noise component of radiation in the RC is determined by the NCE statistics. Case (a) was analyzed in [6]; here we consider case (b) by assuming Gaussian statistics of NCE.

The equation for the Fourier transform, equivalent to Eq. (3) has the form

$$\psi_{N+1}(U) = \int dV \Lambda(U) M(U, V) \psi_N(V), \quad (5)$$

where

$$\psi_N(V) = (2\pi)^{-2} \int dX P_N(X) \exp[-i\text{Re}(XU^*)], \quad (6)$$

$$M(U, V) = (2\pi)^{-2} \int dY \exp[i\text{Re}(YV^*) - i\text{Re}(F(Y)U^*)], \quad (7)$$

$\Lambda(U) = \exp(-R|U|^2/4)$ is the characteristic function of Gaussian noise. Approximation (4) has an equivalent in terms of Fourier transforms

$$M(U, V) \rightarrow \tilde{M}(U, V) = (2\pi|V|)^{-1} \exp(-i\text{Re}U) \delta(|V| - \kappa|U|). \quad (8)$$

By substituting (8) into Eq. (5), we obtain an approximate iteration equation for the characteristic function; after the substitution $\psi_{N+1}, \psi_N \rightarrow \psi_{st}$ it becomes an equation for the characteristic function of a stationary (invariant) distribution, whose solution can be represented in the form

$$\psi_{st}(U) = (2\pi)^{-2} \exp(-i\text{Re}U - R|U|^2/4(1 - \kappa^2)) \prod_{l=1}^{\infty} J_0(\kappa^l|U|). \quad (9)$$

Here $J_0(x)$ is the Bessel function; convergence of the infinite product is obvious. For small κ , using Eq. (9) we can find an approximate expression for the invariant distribution density. The roughest approximation is obtained by replacement of the infinite product in the r.h.s. of Eq. (9) by $J_0(\kappa|U|)$; in this case performing the inverse Fourier transform we arrive at

$$P_{st}(X) \simeq (\pi R)^{-1} (1 - \kappa^2) \exp[-(\kappa^2 + |X - 1|^2)(1 - \kappa^2)/R] I_0(2\kappa(1 - \kappa^2)|X - 1|/R). \quad (10)$$

A more precise expression [having the same precision as that of in the approximate solution of the integral equation for P_{st} with the conditions $R = 0, \kappa \ll 1$, found in [6]; Eq. (19)] is

$$\begin{aligned} P_{st}(X) \simeq & (\pi R)^{-1} (1 - \kappa^2) \exp[-(\kappa^2 + |X - 1|^2)(1 - \kappa^2)/R] \sum_{s=-\infty}^{\infty} (-1)^s I_s(2\kappa^3(1 - \kappa^2)/R) \times \\ & I_s\left(\kappa(1 - \kappa^2)|X - 1|(\sqrt{1 + 2\kappa} + \sqrt{1 - 2\kappa})/R\right) \times \\ & I_s\left(\kappa(1 - \kappa^2)|X - 1|(\sqrt{1 + 2\kappa} - \sqrt{1 - 2\kappa})/R\right). \end{aligned} \quad (11)$$

Here and in Eq. (10), $I_s(x)$ are modified Bessel functions. Distribution P_{st} for small κ is substantially nonzero within a ring with center $X = 1$ and mean radius $|X - 1| \sim \kappa$ and possesses rotational symmetry; a ring shaft with or without a groove in the middle is its graphical representation. Graphs of the distributions in the radial cross sections found numerically are shown in Fig. 1.

The case $T \sim \tau_c \gg \tau^*$ is more difficult for analysis, since one must take into account the effect of correlations in the superposition and interference of light waves. We assume that NCE is a stationary Markovian process; multipoint distributions for NCE can be represented in this case in the form

$$P(\xi_n[t_n], \dots, \xi_0[t_0]) = w(\xi_0) \prod_{p=0}^{n-1} w(\xi_{p+1}, \xi_p, [t_{p+1} - t_p]), \quad (12)$$

where $w(\xi) = \lim_{t \rightarrow \infty} w(\xi, \eta, [t])$; and instants of time are shown in square brackets. Evolution of the signal in this system (in a discrete time scale) can be described by the Kolmogorov-Chapman type equation for a multipoint distribution density

$$P_{N+1}((X_s), \xi_{n+1}) = \int d\mathbf{Y} d\xi P_N((Y_s), \xi_0) \prod_{p=0}^n \delta^{(2)}(X_p - \xi_p - F(Y_p)) w(\xi_{p+1}, \xi_p, [\tau_{p+1} - \tau_p]), \quad (13)$$

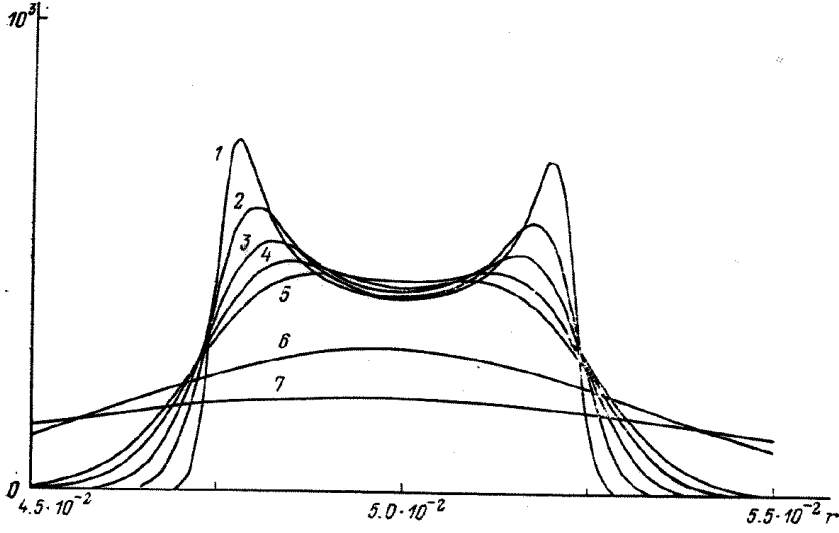


Figure 1: Radial stationary amplitude distributions for $\kappa = 0.05$ and different noise intensities. Here, $r = |X - 1|$; the curves correspond to the following values of R : 1 – 8×10^{-8} , 2 – 3.2×10^{-7} , 3 – 7.2×10^{-7} , 4 – 1.28×10^{-6} , 5 – 2×10^{-6} , 6 – 1.8×10^{-5} , 7 – 5×10^{-5} .

which is the natural generalization of Eq. (3). In Eq. (13) we have used the short notation for the multipoint distribution density; it is expanded as follows

$$P_N((X_s), \xi) = P((X_0[NT_r], X_1[NT_r + \tau_1], \dots, X_n[NT_r + \tau_n]), \xi[NT_r]). \quad (14)$$

Here, first $n + 1$ arguments denote radiation in the RC at different instants of time; the last argument denote the NCE (time values are shown in brackets). In Eq. (13) and below we use notation $d\mathbf{A} = dA_0 dA_1 dA_2 \dots$, and it is assumed that $0 = \tau_0 < \tau_1 < \dots < \tau_n < \tau_{n+1} = T_r$. Applying a generalization of Eq. (5) to the multivariate case to the densities (14) to obtain their Fourier transforms, we can find the evolution equation for the multipoint characteristic functions

$$\psi_{N+1}((U_s), \Omega_{n+1}) = \int d\mathbf{V} d\Omega \psi_N((V_s), \Omega_0) \prod_{p=0}^n M(U_p, V_p) H(\Omega_{p+1}, \Omega_p - U_p, [\tau_{p+1} - \tau_p]), \quad (15)$$

where

$$H(\Omega, \Gamma, [\tau]) = (2\pi)^{-2} \int d\xi d\eta w(\xi, \eta, [\tau]) \exp[i\text{Re}(\eta\Gamma^*) - i\text{Re}(\xi\Omega^*)] \quad (16)$$

and the function $M(U, V)$ is determined by Eq. (7) [the short form of the characteristic function may be expanded using the rule similar to Eq. (14)].

We consider in this paper two models for the statistics of fluctuations, assuming that NCE is a) a stationary Gaussian Markovian random process (GRP), b) a generalized telegraphic Markovian random process (GTRP). The corresponding expressions for the transition probability densities and their Fourier transforms have the form in the GRP case

$$w_G(\xi, \eta, [\tau], R) = \frac{1}{\pi R(1 - \theta_\tau^2)} \exp\left(-\frac{|\xi - \eta\theta_\tau|^2}{R(1 - \theta_\tau^2)}\right), \quad (17)$$

$$H_G(\Omega, \Gamma, [\tau], R) = \delta^{(2)}(\Gamma - \Omega\theta_\tau) \exp\left(-\frac{|\Omega|^2 R}{4}(1 - \theta_\tau^2)\right); \quad (18)$$

in the GTRP case:

$$w_{GT}(\xi, \eta, [\tau]) = \theta_\tau \delta^{(2)}(\xi - \eta) + p(\xi)(1 - \theta_\tau), \quad (19)$$

$$H_{GT}(\Omega, \Gamma, [\tau]) = \theta_\tau \delta^{(2)}(\Omega - \Gamma) + (1 - \theta_\tau) \chi(\Omega) \delta^{(2)}(\Gamma), \quad (20)$$

where

$$p(\xi) = \sum_{k=1}^L p_k \delta^{(2)}(\xi - \xi_k), \quad \chi(\Omega) = \sum_{k=1}^L p_k \exp(-i\text{Re}(\xi_k \Omega^*)),$$

$\theta_\tau = \exp(-\tau/\tau_c)$ (the GTRP is characterized by random jumps in the NCE amplitude between the complex values $\xi_k, k = 1, 2, \dots, L$; the probability for the appearance of ξ_k is p_k). The equation derived from Eq. (15) after application of the random phase approximation (8) will play a central role. We write two versions of this equation, corresponding to different assumptions on the NCE statistics: for the GRP

$$\begin{aligned} \psi_{N+1}((U_s), U_{n+1}) = & \exp\left(-\frac{i}{2} \sum_{p=0}^n [(U_p + U_p^*) - \Lambda((U_s), U_{n+1})] \times \right. \\ & \left. \left\langle \psi_N\left((\kappa U_s e^{i\phi_s}), \sum_{q=0}^{n+1} e^{-\tau_q/\tau_c} U_q\right) \right\rangle_{\text{phase}} \right), \end{aligned} \quad (21)$$

where

$$\Lambda((U_s), U_{n+1}) = \frac{1}{4} R \sum_{p=1}^{n+1} \left| \sum_{q=p}^{n+1} e^{(\tau_p - \tau_q)/\tau_c} U_p \right|^2 (1 - e^{2(\tau_{p-1} - \tau_p)/\tau_c});$$

for the GTRP

$$\begin{aligned} \psi_{N+1}((U_s), U_{n+1}) = & \exp\left(-\frac{i}{2} \sum_{p=0}^n (U_p + U_p^*) - \frac{T_r}{\tau_c}\right) \left[\langle \psi_N((\kappa U_s e^{i\phi_s}), \beta_0^{n+2}) \rangle_{\text{phase}} \times \right. \\ & \sum_{\alpha=1}^{n+1} \sum_{(k_1, \dots, k_\alpha)} \prod_{p=k_1, \dots, k_\alpha} \left(e^{\frac{\tau_p - \tau_{p-1}}{\tau_c}} - 1 \right) \times \\ & \left. \prod_{\gamma=1}^{\alpha} \chi(\beta_{k_\gamma}^{k_\gamma+1}) \langle \psi_N((\kappa U_s e^{i\phi_s}), \beta_0^{k_1}) \rangle_{\text{phase}} \right], \end{aligned} \quad (22)$$

where

$$\beta_p^q = \sum_{i=p}^{q-1} U_i; \quad 1 \leq k_1 < \dots < k_\alpha < k_{\alpha+1} \equiv n+2; \quad \tau_0 = 0; \quad \tau_{n+1} \equiv T_r;$$

averaging is performed over all the angles $\phi_s, s = 1, 2, \dots, n$.

The characteristic functions are generating functions for the multipoint moment functions (MMF); differentiating Eqs. (21,23) we can construct an evolution mapping for MMF. We define the MMF as

$$\begin{aligned} m_N(p, q, (k_s, l_s)) = & \left\langle \xi^p \xi^{*q} \prod_{i=0}^n Z_i^{k_i} Z_i^{*l_i} \right\rangle_N = \\ (2\pi)^{2n+4} (2i)^D & \left(\frac{\partial^p}{\partial \Omega^{*p}} \frac{\partial^q}{\partial \Omega^q} \right) \prod_{i=0}^n \left(\frac{\partial^{k_i}}{\partial U^{*k_i}} \frac{\partial^{l_i}}{\partial U^{l_i}} \right) \psi_N((U_s), \Omega) \Big|_{U_s=0}^{\Omega=0}. \end{aligned} \quad (23)$$

Here

$$D = p + q + \sum_{i=1}^n (k_i + l_i)$$

is the order of the MMF. We call the MMF (24) the N -moments, i.e., the mean of one-term polynomials over distribution (13) formed by signal amplitudes Z_s (taken at the time values $NT_r + \tau_s$, $s = 0, 1, \dots, n$) and also by the NCE amplitude (at NT_r). The structure of Eqs. (21,23) is such that any $(N + 1)$ -moment of a certain order is expressed linearly in terms of the N -moments of the same or lower orders. For this reason, evolution mappings for the MMF are linear and finite-dimensional. We note that, if we do not employ the approximation (8), the $(N + 1)$ -moment is expressed in terms of the N -moments of all orders, and the finite-dimensional mapping for the MMF can be obtained only by using some uncoupling procedure (see, for example, [13]).

We can assume that radiation dynamics leads to the formation of a limiting stationary (invariant) random process in the limit $N \rightarrow \infty$ (this corresponds to an instant of time, substantially larger than T_r). By confining ourselves to the treatment of GRP- and GTRP-type NCE, we find that evolution mapping of the MMF, which describes dynamics of a finite (but arbitrarily large) set of MMF, including all MMF of order lower than given order, possesses a stable stationary point. We first note that the matrix \mathbf{A} in the evolution mapping $\mathbf{M}_{N+1} = \mathbf{A}\mathbf{M}_N + \mathbf{B}$, can be made triangular by a special choice of the basis in the linear MMF space; \mathbf{M}_N is the vector of the finite-dimensional space of N -moments. In fact, Eqs. (21,23) are such that $(N + 1)$ -moment of order D , characterized by given value $d = p + q$, is a linear combination of moments of order $D' \leq D$, in which only N -moments with $D' = D$ with $d' \geq d$ appear. Taking this into account for the construction of the basis, in which matrix \mathbf{A} is triangular, it is necessary to introduce an indexing for the MMF in such a manner that moments with larger D have larger indices and among moments with given D , those with smaller d have larger indices. The diagonal elements of matrix \mathbf{A} (its eigenvalues) are equal for the GRP

$$A(p, q, (k_s, l_s)) = \prod_{i=0}^n \delta_{k_i, l_i} \kappa^{k_i + l_i} e^{T_r(p+q)/\tau_c}, \quad (24)$$

and for the GTRP

$$A(p, q, (k_s, l_s)) = \prod_{i=0}^n \delta_{k_i, l_i} \kappa^{k_i + l_i} (\delta_{p_0} \delta_{q_0} + e^{T_r/\tau_c} (1 - \delta_{p_0} \delta_{q_0})), \quad (25)$$

(each of these is strictly less than unity). Hence, the equation $\mathbf{M}_{st} = \mathbf{A}\mathbf{M}_{st} + \mathbf{B}$ for a stationary point in the MMF space is equivalent to a system of linear algebraic equations with a nonsingular matrix and, therefore, has a unique solution. To show the stability of the stationary point, we write the solution of the linearized equation for small deviation: $\delta\mathbf{M}_{N+1} = \mathbf{A}\delta\mathbf{M}_N$ representing \mathbf{A} in the normal Jordan form [14]:

$$\delta\mathbf{M}_N = \mathbf{A}^N \delta\mathbf{M}_0 = (\mathbf{T}\mathbf{J}\mathbf{T}^{-1})^N \delta\mathbf{M}_0 = \mathbf{T}\mathbf{J}^N \mathbf{T}^{-1} \delta\mathbf{M}_0. \quad (26)$$

Here, \mathbf{J} is the direct sum of Jordan blocks (having the same diagonal elements as those of \mathbf{A}), and \mathbf{T} is some nonsingular matrix. If the diagonal elements of matrix \mathbf{J} are less than unity, $\mathbf{J}^N \rightarrow 0$ is satisfied as $N \rightarrow \infty$ (see [14]). It follows from Eq. (26) that $\delta\mathbf{M}_N \rightarrow 0$ as $N \rightarrow \infty$, so that the stationary point is stable.

Together with multipoint functions of type (13) correspond to points on the time axis, located inside a segment of length T_r , it is necessary to be able to find common-type functions.

We can show that it is not necessary to reconstruct these functions and solve the evolution equations for them, since any such function for points on the time axis, spaced larger than T_r , is expressed in terms of the functions considered above. The general expressions are cumbersome, so we do not give their explicit expressions.

By solving the set of linear equations of the MMF, corresponding to a stationary point, we can find any MMF in an explicit form. Let us consider a number of examples. We note that, in the random phase approximation, phase correlation vanishes after single pass of the wave through the nonlinear element. It is instructive to calculate intensity correlation corresponding to different instants of time. We denote

$$C(\tau) = \langle |Z_{t+\tau}|^2 |Z_t|^2 \rangle_{\text{st}} - \langle |Z_{t+\tau}|^2 \rangle_{\text{st}} \langle |Z_t|^2 \rangle_{\text{st}} \quad (27)$$

the covariance of radiation intensities at a fixed point of the RC. For the GRP

$$C(\tau) = \frac{1}{1 - \kappa^4} \left(R^2 \frac{\theta_\tau^2 + \kappa^2(\theta/\theta_\tau)^2}{1 - \kappa^2\theta^2} + 2R \frac{\theta_\tau + \kappa^2(\theta/\theta_\tau)}{1 - \kappa^2\theta} \right), \quad (28)$$

$$C(kT_r + \tau) = \kappa^{2k} C(\tau) + 2R \frac{\theta\theta_\tau}{1 - \kappa^2\theta} \frac{\kappa^{2k} - \theta^k}{\kappa^2 - \theta} + R^2 \frac{\theta^2\theta_\tau^2}{1 - \kappa^2\theta^2} \frac{\kappa^{2k} - \theta^{2k}}{\kappa^2 - \theta^2}, \quad (29)$$

and for the GTRP

$$C(\tau) = \frac{\theta_\tau + \kappa^2(\theta/\theta_\tau)}{(1 - \kappa^4)(1 - \kappa^2\theta)} (\langle Q^2 \rangle_{GT} - \langle Q \rangle_{GT}^2), \quad (30)$$

$$C(kT_r + \tau) = \kappa^{2k} C(\tau) + \frac{\theta\theta_\tau}{1 - \kappa^2\theta} \frac{\kappa^{2k} - \theta^k}{\kappa^2 - \theta} (\langle Q^2 \rangle_{GT} - \langle Q \rangle_{GT}^2), \quad (31)$$

where

$$\theta = \exp(-T_r/\tau_c), \quad \theta_\tau = \exp(-\tau/\tau_c), \quad 0 < \tau < T_r, \quad Q = 1 + \xi + \xi^* + |\xi|^2;$$

averaging of the GTRP with a single-point distribution density is denoted by the symbol $\langle \dots \rangle_{GT}$.

Let us compare the found expressions with their analogs, corresponding to a RC with a linear absorber. The method of computing these expressions do not differ from the above mentioned, if we use Eq. (15), in which the kernel (7) corresponds to the mapping $F(Z) = 1 + \kappa Z e^{i\delta}$. For RC with a linear absorber and GRP-type NCE from Eq. (15) we can find the multipoint distribution functions in explicit form:

$$P_{\text{st}}^{\text{lin}}((X), \xi) = w_G(\xi, R) w_G(X - \tilde{X}, \kappa(X - \tilde{X})e^{i\delta} + \xi, [T_r], \frac{R}{1 - \kappa^2}) |1 - \kappa e^{i\delta} e^{-T_r/\tau_c}|^2 \quad (32)$$

$$P_{\text{st}}^{\text{lin}}((X, Y), \xi) = w_G(\xi, R) w_G(Y - \tilde{X}, X - \tilde{X}, [\tau], \frac{R}{1 - \kappa^2}) \times \\ w_G(X - \tilde{X}, \kappa(Y - \tilde{X})e^{i\delta} + \xi, [T_r - \tau], \frac{R}{1 - \kappa^2}) |1 - \kappa e^{i\delta} e^{-T_r/\tau_c}|^2 \quad (33)$$

where $\tilde{X} = (1 - h)^{-1}$, $h = \kappa e^{-i\delta}$ [single- and two-point GRP distribution densities (17) are used in writing the r.h.s.]. We can find an expression for the intensity covariance directly from Eqs. (32,33)

$$C(kT_r + \tau) = R|1 - h|^{-2} (\Gamma_k + \Gamma_k^*) + R^2 |\Gamma_k|^2, \quad (34)$$

$$\Gamma_k = \frac{h^{*k}}{1 - |h|^2} \left(\frac{\theta_\tau}{1 - h\theta} - \frac{h^*(\theta/\theta_\tau)}{1 - h^*\theta} \right) + \frac{\theta\theta_\tau}{1 - h\theta} \frac{h^{*k} - \theta^k}{h^* - \theta}. \quad (35)$$

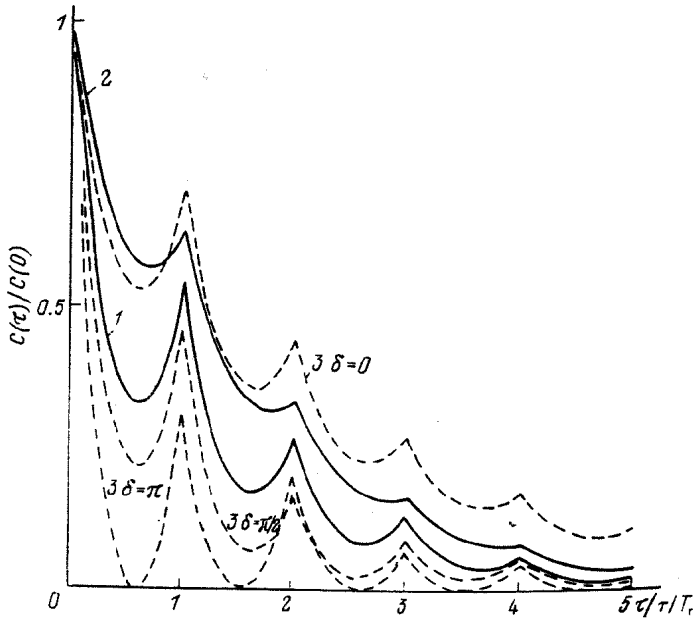


Figure 2: Intensity correlation functions for $\kappa = 0.7, \tau_c/T_r = 0.6$; curves 1 and 2 are for a RC with a nonlinear medium and NCE: 1 – GRP type, $R = 9.2$, 2 – GTRP type; curves 3 correspond to RC with a linear absorber with different δ and GRP-type NCE.

Graphs of the correlation functions are given in Fig. 2.

We conclude by noting that we can extend the results obtained to other nonlinear oscillatory systems with retardation, in which the oscillations may have a different physical nature (for example, hybrid optoelectronic or radioelectronics devices [7, 8, 4]). It is most important from a physics point of view that the ability to convert a coherent into a noise signal and to transform the statistical characteristics of the noise be the common feature of such systems.

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