

Absorptive multistability in a system of three level centers

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A phenomenon of optical bistability draws attention as important and simple example of nonequilibrium phase transition (generation of a "coherent state" dissipative structure) in nonlinear active medium [1], and also in relation to possible technical applications like optical memory cells, optical amplifiers, cutoffs, etc., see [2]. Existence of the bistable regime in two-level media with absorptive nonlinearity is experimentally confirmed in two cases: (a) in exact frequency resonance of external driving field and atomic transition [2], (b) for nonzero frequency detuning [4]. We show that in a system of three-level centers interacting with two modes of the external field even in a simplest case of purely absorptive nonlinearity (under exact resonance condition and neglecting all other types of nonlinearities) one can observe not only bistability but even more complicate nonequilibrium critical phenomena like three- and tetrastability and multiple transitions. Other types of nonlinearities leading to multistability were discussed in [5]-[7].

Consider an active medium of N three-level centers inside the bimodal cavity excited by the external driving beam through the semitransparent mirror. We assume that intensities of optical oscillations corresponding to different cavity modes are controlled independently. Denote the basic state of ν -th active center as $|0, \nu\rangle$, the excited states of the same atom are $|\pm, \nu\rangle$. The Hamiltonian of the system is defined as

$$H = i\hbar \sum_{\nu=1}^N \sum_{I=\pm} g_I (E_I^* |0, \nu\rangle \langle I, \nu| - E_I |I, \nu\rangle \langle 0, \nu|), \quad (1)$$

where E_I denotes complex amplitude of I -th mode in dimensionless units and g_I is the interaction coefficient of I -th mode ($I = \pm$). Introduce averaged polarizations and population inversions (normalized to the number of active centers), which expressed through the elements of one-particle density matrix:

$$p_{\pm} = N^{-1} \sum_{\nu=1}^N \rho_{0\pm}^{(\nu)}, \quad d_{\pm} = N^{-1} \sum_{\nu=1}^N \rho_{\pm\mp}^{(\nu)}, \quad n_{\pm} = N^{-1} \sum_{\nu=1}^N (\rho_{\pm\pm}^{(\nu)} - \rho_{00}^{(\nu)}). \quad (2)$$

Using semiclassical approximation and mean field approximation (applicability field of the latter is discussed for example in [8]), we obtain the system of equations:

$$\begin{aligned} \dot{n}_{\pm} &= -(2g_{\pm} E_{\pm} p_{\pm} + g_{\mp} E_{\mp} p_{\mp} + c.c.) - \tau_{\parallel}^{-1} (n_{\pm} + 1) \mp T_{\parallel}^{-1} (n_{+} - n_{-}), \\ \dot{p}_{\pm} &= g_{\pm} E_{\pm}^* d_{\pm} + g_{\mp} E_{\mp}^* d_{\mp} - \tau_{\perp}^{-1} p_{\pm}, \\ \dot{d}_{\pm} &= -g_{\pm} E_{\pm} p_{\mp} - g_{\mp} E_{\mp}^* p_{\pm}^* - T_{\perp}^{-1} d_{\pm}, \\ \dot{E}_{\pm} &= N g_{\pm} p_{\pm}^* - k_{\pm} (E_{\pm} - E_{\pm}^0), \end{aligned} \quad (3)$$

where we add to the r.h.s. of equations the relaxation terms containing phenomenological relaxation times $\tau_{\perp}, \tau_{\parallel}$ and T_{\perp}, T_{\parallel} for transitions $(0) \Leftrightarrow (\pm)$ and $(+) \Leftrightarrow (-)$ respectively. We

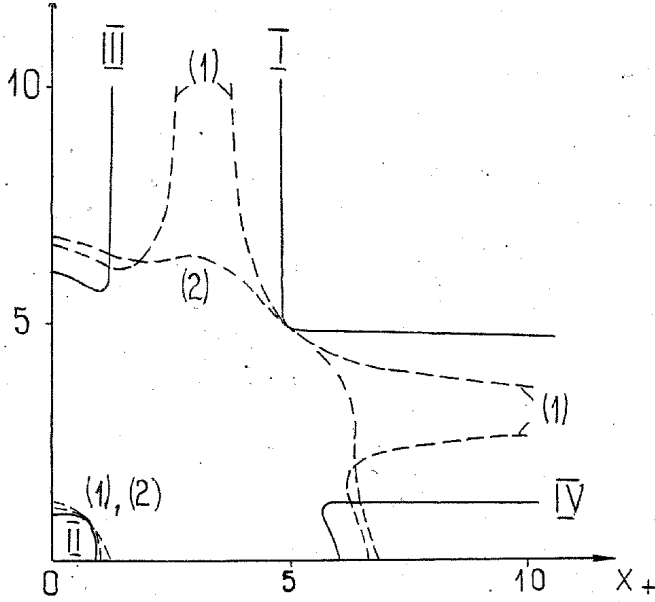


Figure 1: .

introduced notations: k_I for relaxation constant for I -th mode of optical field in the cavity, E_I^0 for the steady state value of the I -th mode in absence of the active media. In order to simplify the problem, we assume $T_{\perp} \ll T_{\parallel}, \tau_{\perp}, \tau_{\parallel}$ (coherence in the excited state returns to the basic value very fast). In steady state regime the system (4) reduces to:

$$y_{\pm} = x_{\pm}(1 + 4C_{\pm}S_{\mp}/P), \quad (4)$$

where

$$S_{\pm} = |x_{\pm}|^2 + 2(1 + 2\Gamma), \quad P = 3S_+S_- - 2(1 + 3\Gamma)(S_+ + S_-), \quad \Gamma = \tau_{\parallel}/T_{\parallel}, \\ C_{\pm} = g_{\pm}^2 N \tau_{\perp} / 2k_{\pm}, \quad x_{\pm} = 2g_{\pm} E_{\pm} \sqrt{\tau_{\parallel} \tau_{\perp}}, \quad y_{\pm} = 2g_{\pm} E_{\pm}^0 \sqrt{\tau_{\parallel} \tau_{\perp}}.$$

From (4) it follows that $\arg x_{\pm} = \arg y_{\pm}$, so that x_{\pm}, y_{\pm} can be assumed to be real and positive. System (4) has up to four stable solutions denoted as A, B, C, D ; stability boundaries are show in Figs. 1 and 2 ($C_{\pm} = 20$). At $\Gamma = 0$ (solid lines) stability regions are located in the first quadrant: A is to the right and above the line I, B is to the left and below the line II, C is to the left and above the line III, D is to the right and below the line IV.

As it is seen from Fig. 1, at different sets of parameters y_+, y_- there exist from 1 to 4 stable solutions. When the parameters are varied (by changing of external excitation intensities) some solutions became unstable, and system undergoes drasic changes. Boundaries of stability regions represent a bifurcation set generated by projections of the fold and cusp catastrophe points onto the plane (y_+, y_-) [9]. Dashed lines (1) and (2) in fig. 1 correspond to $\Gamma = 1$ and $\Gamma = 2$; number of stable states is equal to two for $\Gamma > 1$. Note that detailed description of switch dynamics in this model requires to go beyond the semiclassical approximation and considering nonstationary regime (for the two-level model it was discussed in [10]). The stationary stable states are characterized by large population differencies of (+) and (-) levels, which leads to nonzero stationary polarization in the excited state (or magnetization, if the excited levels are Zeeman sublevels; see [6]).

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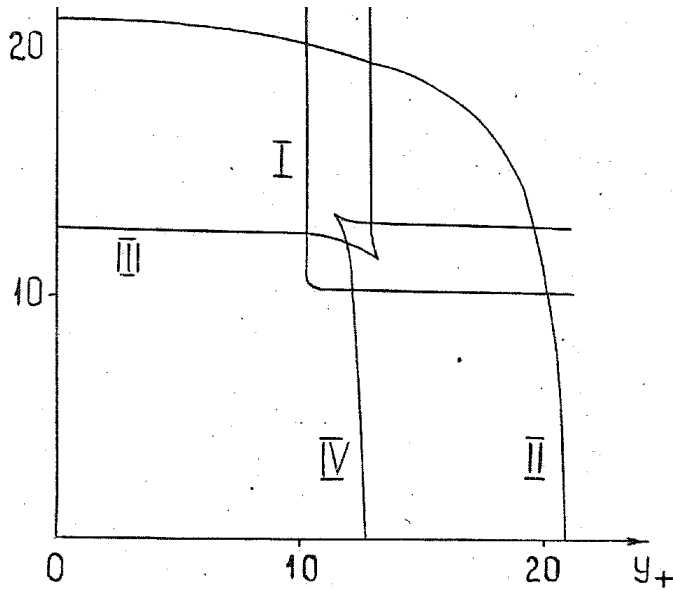


Figure 2: .

References

- [1] Bonifacio R., and Lugiato L.A., in *Pattern Formation in Dynamical Systems and Pattern Recognition*, Springer Series in Synergetics, **5**, part. II, 16 (1979).
- [2] Lugovoy V.N., *Kvantovaya Elektronika*, **6**, 2053 (1979) [in Russian].
- [3] Weyer K.G., Wiedenmann H., Rateike M., McGullivray W.R., Meystre P., and Walter H., *Opt.Comm.* **37**, 426 (1981).
- [4] Gibbs H.M., McCall S.L., and Venkatesan T.N.C., *Phys.Rev.Lett.* **36**, 1135 (1976).
- [5] Perel'man I.F., Kovarskii V.A., and Averbuch I.Sh., *JETP*, **80**, 80 (1981) [in Russian].
- [6] Kitano M., Yabuzaki T., and Ogava T., *Phys.Rev.* **A24**, 3156 (1981).
- [7] Agrawal G.P., and Flytzanis C., *Phys.Rev.* **A24**, 3173 (1981).
- [8] Bonifacio R., and Lugiato L.A., *Letts. Nuovo Cimento*, **21**, 505 (1978).
- [9] Poston T. and Steward I. *Theory of Catastrophes and its Applications*, Moscow, Mir Publishers (1980) [in Russian].
- [10] Farina J.D., Narducci L.M., Yuan J.M., and Lugiato L.A., *Optical Engineering*. **19**, 469 (1980).